

MEASURING THE SPIN OF SPIRAL GALAXIES

C. TONINI¹, A. LAPI^{1,2}, F. SHANKAR¹, P. SALUCCI¹*Draft version February 5, 2008*

ABSTRACT

We compute the angular momentum, the spin parameter and the related distribution function for Dark Matter halos hosting a spiral galaxy. We base on scaling laws, inferred from observations, that link the properties of the galaxy to those of the host halo; we further assume that the Dark Matter has the same total specific angular momentum of the baryons. Our main results are: (i) we find that the gas component of the disk significantly contributes to the total angular momentum of the system; (ii) by adopting for the Dark Matter the observationally supported Burkert profile, we compute the total angular momentum of the disk and its correlation with the rotation velocity; (iii) we find that the distribution function of the spin parameter λ peaks at a value of about 0.03, consistent with a no-major-merger scenario for the late evolution of spiral galaxies.

Subject headings: galaxies: halos - galaxies: spiral - galaxies: formation - galaxies: kinematics and dynamics

1. INTRODUCTION

The mechanism of galaxy formation, as currently understood, involves the cooling and condensation of baryons inside the gravitational potential well provided by the Dark Matter (DM); in spirals, a rotationally supported disk is formed, whose structure is governed by angular momentum acquired through tidal interactions during the precollapse phase.

Under the assumption of specific angular momentum conservation, that holds when the baryons and the DM are initially well mixed, the dynamics of the dark halo is directly related to the disk scale length (see Fall & Efstathiou 1980). This tight connection between halo dynamics and disk geometry is quantified by the spin parameter λ (Peebles 1969).

The general procedure for the computation of the angular momentum has been described in detail by Mo, Mao & White (1998); it relies on 3 basic assumptions: (i) the mass of the galactic disk is a universal fraction of the halo's; (ii) the total angular momentum of the disk is also a fixed fraction of the halo's; (iii) the disk is thin and centrifugally supported, with an exponential surface density profile. The theory is applied to a Navarro, Frenk & White (1997; NFW) DM potential.

In the present work, we propose to determine the angular momentum and the spin parameter of disk galaxies by making use of the observed matter distribution in spirals, and of observed scaling relations between halo and disk properties. For this purpose, we adopt a modified set of assumptions: we relax (i), and use instead an empirical relation that links the disk mass to that of its DM halo (Shankar et al. 2005); we retain (ii) and suppose total specific angular momentum conservation during the disk formation, i.e., $J_D/M_D = J_H/M_H$ in terms of the disk and halo masses M_D , M_H and of the related total angular momenta J_D , J_H ; as to (iii), we still assume that the disk is centrifugally supported, stable, and distributed according to an exponential surface density profile, but we also take into account the gaseous (HI+He) component. Finally, we perform the computation for a Burkert halo.

We adopt a flat cosmology with matter density parameter $\Omega_M \approx 0.27$ and Hubble constant $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} = 0.71$. Given a halo with mass M_H we determine its radius as $R_H = [3M_H \Omega_M^z / 4\pi \rho_c \Omega_M (1+z)^3 \Delta_H]^{1/3}$; here $\rho_c =$

$2.8 \times 10^{11} h^2 M_\odot \text{ Mpc}^{-3}$ is the critical density, $\Omega_M^z = \Omega_M (1+z)^3 / [(1-\Omega_M) + \Omega_M (1+z)^3]$ and $\Delta_H = 18\pi^2 + 82(\Omega_M^z - 1) - 39(\Omega_M^z - 1)^2$ are the density parameter and the density contrast at redshift z ; $\Delta_H \approx 100$ holds at $z = 0$.

2. THE ANGULAR MOMENTUM

The fundamental parameters of the stellar disk and halo mass distributions can be obtained straightforwardly by means of three observational scaling relations, linking the disk mass to the halo mass, to the halo central density, and to the disk scale length.

The total mass of the stellar disk M_D that resides in a halo of mass M_H is given by the relation (Shankar et al. 2005):

$$M_D \approx 2.3 \times 10^{10} M_\odot \frac{(M_H/3 \cdot 10^{11} M_\odot)^{3.1}}{1 + (M_H/3 \cdot 10^{11} M_\odot)^{2.2}}; \quad (1)$$

this holds for halo masses between 10^{11} and about $3 \times 10^{12} M_\odot$, wide enough to include most of the spiral population, except dwarfs. This relation has been derived by the statistical comparison of the galactic halo mass function with the stellar mass function; the related uncertainty is around 20%, mainly due to the mass to light ratio used to derive the stellar mass function from the galaxy luminosity function.

We model the stellar disk with a thin, exponential surface density profile of the form

$$\Sigma_D(r) = \frac{M_D}{2\pi R_D^2} e^{-r/R_D}. \quad (2)$$

The characteristic scale-length R_D is estimated through:

$$\log \frac{R_D}{\text{kpc}} = 0.633 + 0.379 \log \frac{M_D}{10^{11} M_\odot} + 0.069 \left(\log \frac{M_D}{10^{11} M_\odot} \right)^2; \quad (3)$$

this relation is inferred from dynamical mass determinations by Persic et al. (1996). These scale lengths are consistent with the data by Dale et al. (1999), Simard et al. (1999), and Courteau et al. (2003).

For the DM, we adopt a Burkert distribution $\rho_H(r) = \rho_0 R_0^3 / (r + R_0)(r^2 + R_0^2)$, with R_0 the core radius and ρ_0 the effective core density. Correspondingly, the total halo mass inside the radius r is given by

$$M_H(< r) = 4M_0 \left[\ln \left(1 + \frac{r}{R_0} \right) - \tan^{-1} \left(\frac{r}{R_0} \right) + \frac{1}{2} \ln \left(1 + \frac{r^2}{R_0^2} \right) \right], \quad (4)$$

¹ Astrophysics Sector, SISSA/ISAS, Via Beirut 2-4, I-34014 Trieste, Italy

² Univ. "Tor Vergata", Via Ricerca Scientifica 1, I-00133 Roma, Italy

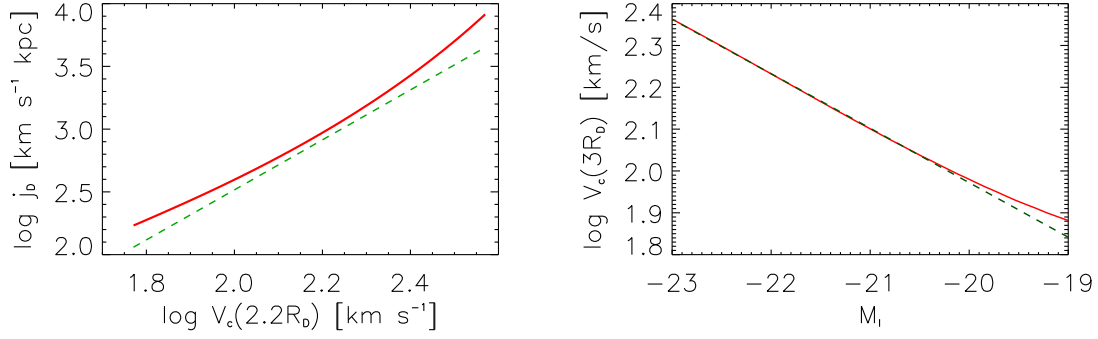


FIG. 1.— Left panel: the specific angular momentum of the disk as a function of the rotation velocity at $2.2R_D$. *Solid* line is the result from this work, adopting the Burkert profile; *dashed* line is the best-fit relation from the data collected by Navarro & Steinmetz (2000), see their Figure 3. Right panel: the Tully-Fisher relation. *Solid* line represents the result from this work and *dashed* line illustrates the fit to the data by Giovanelli et al. (1997).

with $M_0 = 1.6\rho_0 R_0^3$ being the mass contained inside the radius R_0 . The density ρ_0 is determined from the disk mass through the relation obtained from the Universal Rotation Curve (Burkert & Salucci 2000):

$$\log \frac{\rho_0}{\text{g cm}^{-3}} = -23.515 - 0.964 \left(\frac{M_D}{10^{11} M_\odot} \right)^{0.31}. \quad (5)$$

For each given virial mass M_H (corresponding to a radius R_H) we find the density ρ_0 through Eqs. (1) and (5); then we numerically compute the core radius R_0 by requiring that the mass $M_H(< R_H)$ inside R_H given by r.h.s. of Eq. (4) equals the virial mass M_H . The resulting relation R_0 vs. M_H is approximated within a few percents by the relation

$$\log(R_0/\text{kpc}) \approx 0.66 + 0.58 \log(M_H/10^{11} M_\odot); \quad (6)$$

such values of R_0 obtained by assuming a mass model out to R_H are consistent with those estimated by Burkert & Salucci (2000) from the decomposition of the inner rotation curves.

The total circular velocity of the disk system is $V_c^2(r) = V_D^2(r) + V_H^2(r)$. For a thin, centrifugally supported disk the circular velocity is given by $V_D^2(r) = (GM_D/2R_D) x^2 B(x/2)$; here $x = r/R_D$ and the quantity $B = I_0 K_0 - I_1 K_1$ is a combination of the modified Bessel functions that accounts for the disk asphericity. Moreover, the halo circular velocity is simply $V_H^2(r) = GM_H(< r)/r$, and it is useful to define $V_H = \sqrt{GM_H/R_H}$. Given the relations (1), (3), (5) and (6) linking the basic quantities of the system, the shape and amplitude of the velocity profile depend only on the halo mass. Note that the uncertainties on these relations combine to give a 10%–20% uncertainty on the determination of the velocity profile (see Tonini & Salucci 2004).

In order to check our mass model and empirical scaling relations, we compute the I-band Tully-Fisher relation at $r = 3R_D$. We obtain the B-band luminosity from the stellar disk mass through the relation $\log(L_B/L_\odot) \approx 1.33 + 0.83 \log(M_D/M_\odot)$ by Shankar et al. (2005), then convert the related magnitude in I-band through the mean colour $B-I \approx 2$ (Fukugita et al. 1995). In Figure 1 (right) we compare the result with the data by Giovanelli et al. (1997), finding an excellent agreement.

We compute the angular momentum of the disk as

$$J_D = 2\pi \int_0^\infty \Sigma_D(r) r V_c(r) r dr = M_D R_D V_H f_R \quad (7)$$

with $f_R = \int_0^\infty x^2 e^{-x} V_c(xR_D)/V_H dx$, $x = r/R_D$ and $M_D = 2\pi \Sigma_0 R_D^2$. Note that J_D depends linearly both on the mass and on the radial extension of the disk, while the DM distribution enters the computation through the integrated velocity profile, encased into the shape factor f_R ; the latter slowly varies (by a factor 1.3 at most) throughout our range of halo masses.

In Figure 1 (left) we show the specific angular momentum of the disk vs. the total circular velocity at $r = 2.2R_D$, computed as $j_D = J_D/M_D$ from Eq. (7). Plotted for comparison is also the best-fit relation by Navarro & Steinmetz (2000) from their collection of data; note that these authors adopted a flat rotation curve, so that $f_R = 2$ and $j_D = 2R_D V_H$.

We derive the halo angular momentum by assuming the conservation of the total specific angular momentum between DM and baryons:

$$J_H = J_D \frac{M_H}{M_D}, \quad (8)$$

an *ansatz* widely supported/adopted in the literature (Mo et al. 1998; van den Bosch et al. 2001, 2002; Burkert & D’Onghia 2004; Peirani et al. 2004). Note that small variations of J_D are magnified by a factor M_H/M_D in the value of J_H , i.e., the latter is rather sensitive to the radial extension of the baryons.

We now consider, along with the stars, the gaseous component that envelops the disk of spiral galaxies. We derive the total mass of the gas component from the disk luminosity (see above) through the relation

$$M_{\text{gas}} = 2.13 \times 10^6 M_\odot \left(\frac{L_B}{10^6 L_\odot} \right)^{0.81} \left[1 - 0.18 \left(\frac{L_B}{10^8 L_\odot} \right)^{-0.4} \right] \quad (9)$$

by Persic & Salucci (1999), where we have included a factor 1.33 to account for the He abundance. Since the gas mass is on average much less than the halo mass, both the total mass and the rotation curve remain virtually unaltered ($V_{\text{gas}} \sim \sqrt{M_{\text{gas}}/R_{\text{gas}}}$) by the presence of the gaseous component. However, the gas is much more diffuse than the stars, reaching out several disk scale lengths (Corbelli & Salucci 2000; Dame 1993); since most of the angular momentum comes from material at large radial distances (van den Bosch et al. 2001), we expect the gas to add a significant contribution to the total angular momentum (Eq. [7]), especially in small spirals where the gas to baryon fraction is close to 50%.

The detailed density profile of the gas in spirals is still under debate in the literature. However, we are confident that the main factors entering the computation of the gas angular momentum J_{gas} are just the gas total mass M_{gas} and the radial extension of its distribution; in other words, we expect that the details of the gas profile do not significantly affect the results. In order to check this statement, we considered 3 different gas models, *i.e.* (i) a disk-like distribution (DL), with scale length αR_D ; (ii) a uniform distribution (U) out to a radius βR_D ; and (iii) an M33-like gaussian distribution (M33; Corbelli & Salucci 2000):

$$\begin{aligned}\Sigma_{\text{gas}}^{\text{DL}}(r) &= \frac{M_{\text{gas}}}{2\pi \alpha^2 R_D^2} e^{-r/\alpha R_D} \\ \Sigma_{\text{gas}}^{\text{U}}(r) &= \frac{M_{\text{gas}}}{\pi \beta^2 R_D^2} \theta(r - \beta R_D) \\ \Sigma_{\text{gas}}^{\text{M33}}(r) &= \frac{M_{\text{gas}}}{\pi (2k_1^2 + k_2^2) R_D^2} e^{-(r/k_1 R_D) - (r/k_2 R_D)^2},\end{aligned}\quad (10)$$

where θ in the second equation is the Heaviside step function. As fiducial values of the parameters, we adopt $\alpha \approx 3$ in the first expression, $\beta \approx 6$ in the second one (Dame 1993), and $k_1 \approx 11.9$, $k_2 \approx 5.87$ in the last one (Corbelli & Salucci 2000). Each profile has been normalized to the total gas mass M_{gas} as computed from Eq. (9).

In parallel with Eq. (7), the gas angular momentum will be

$$J_{\text{gas}} = 2\pi \int_0^\infty \Sigma_{\text{gas}}(r) r V_c(r) r dr = M_{\text{gas}} R_D V_H f_{\text{gas}}, \quad (11)$$

where the shape parameter f_{gas} encodes the specific gas distribution. On comparing its values for the three models we find differences of less than 15%, and so confidently choose the gaussian profile as a baseline.

We then compute the halo angular momentum as a function of the total baryonic one:

$$J_H = (J_D + J_{\text{gas}}) \frac{M_H}{M_D + M_{\text{gas}}}. \quad (12)$$

The gas is dynamically affecting the system mainly through its different spatial distribution with respect to that of the stars, adding an angular momentum component that is significant at large radii compared to R_D .

Note that we do not include a bulge component, since it would contribute a negligible angular momentum and a mass of $0.2 M_D$ at most; in any case, this would slightly lower J_H after Eq. (12) and, as will be evident in the next Section, would lower the spin parameter and strengthen our conclusions.

3. THE SPIN PARAMETER

The spin parameter is a powerful tool to investigate galaxy formation, as it is strictly related to both the dynamics and the geometry of the system. We compute its values and distribution function based on the results of Section 2.

The spin parameter is defined as follows:

$$\lambda = \frac{J_H |E_H|^{1/2}}{G M_H^{5/2}} \quad (13)$$

where G is the gravitational constant, and E_H is the total energy of the halo. The latter is computed as $|E_H| = 2\pi \int dr r^2 \rho_H(r) V_c^2$ after the virial theorem, and having supposed that all the DM particles orbit on circular tracks.

Bullock et al. (2001) proposed the alternative definition

$$\lambda' = \frac{J_H}{\sqrt{2} M_H R_H V_H} = \frac{J_H + J_D + J_{\text{gas}}}{\sqrt{2} (M_H + M_D + M_{\text{gas}}) R_H V_H}, \quad (14)$$

where the second equality holds after Eq. (12). We find that for Burkert halos the ratio λ/λ' is between 1.1–1.3 in the mass range $10^{11} - 3 \times 10^{12} M_\odot$. In Figure 2 (top panels) we plot both λ and λ' as a function of the halo mass. We highlight the difference in the value of the spin parameter when the gas component is included, especially in low mass halos.

To compute the probability distributions $\mathcal{P}(\lambda)$ and $\mathcal{P}(\lambda')$ of the spin parameters, we make use of the galactic halo mass function, *i.e.*, the number density of halos with mass M_H containing a single baryonic core, as derived by Shankar et al. (2005). A good fit is provided by the Schechter function $\Psi(M_H) = (\Psi_0/\bar{M})(M_H/\bar{M})^\alpha \exp(-M_H/\bar{M})$, with parameters $\alpha = -1.84$, $\bar{M} = 1.12 \times 10^{13} M_\odot$ and $\Psi_0 = 3.1 \times 10^{-4} \text{ Mpc}^{-3}$; note that within our range of halo masses, this is mostly contributed by spirals. For the computation of $\mathcal{P}(\lambda)$ or $\mathcal{P}(\lambda')$, we randomly picked up a large sample of masses distributed according to $\Psi(M_H)$, then compute λ or λ' for each using Eqs. (13) and (14), and eventually build up the statistical distributions. During this procedure we have convolved the relations (13) and (14) with a gaussian scatter of 0.15 dex that takes into account the statistical uncertainties in the empirical scaling laws we adopt; these are mostly due to the determination of R_D through Eq. (3), for which we have determined the scatter by using the disk mass estimates of individual spirals reported in Persic & Salucci (1990).

As shown in Figure 2 (bottom panels), we find a distribution peaked around a value of about 0.03 for λ and about 0.025 for λ' , when the gas is considered. We stress that this value of λ' is close to the result of the simulations by D’Onghia & Burkert (2004), who on average find $\lambda' = 0.023$ for spirals quietly evolving (*i.e.*, experiencing no major mergers) since $z \approx 3$, see their Figure 4. In addition, Burkert & D’Onghia (2005) argue that this value of λ' provides a very good fit to the observed relation between the disk scale length and the maximum rotation velocity (see their Figure 1). We also stress that our most probable value for λ is in agreement with the results by Gardner (2001), Vitvitska et al. (2002) and Peirani et al. (2004), who find a peak value of the distribution function at around 0.03 for halos that evolved mainly through smooth accretion.

4. DISCUSSION AND CONCLUSIONS

In this *Letter* we have computed the angular momentum, the spin parameter and the related distribution function for DM halos hosting a spiral galaxy. We have relied on observed scaling relations linking the properties of the baryons to those of their host halos, and have assumed the same total specific angular momentum for the DM and the baryons.

Our main findings are: (i) we show that including the gas component beside the stars has a remarkable impact on the total angular momentum; (ii) by adopting for the DM the observationally supported Burkert profile, we compute the total angular momentum of the disk and its relationship with the rotation velocity; (iii) we obtain $\lambda' \approx 0.025$ and $\lambda \approx 0.03$ as most-probable values of the spin parameters.

Simulations based on the Λ CDM framework, performed by various authors (Bullock et al. 2001; D’Onghia & Burkert 2004), have shown that the distribution of the spin parameter λ' for the whole halo catalogue peaks at around 0.035, significantly higher than our empirical value. However, D’Onghia

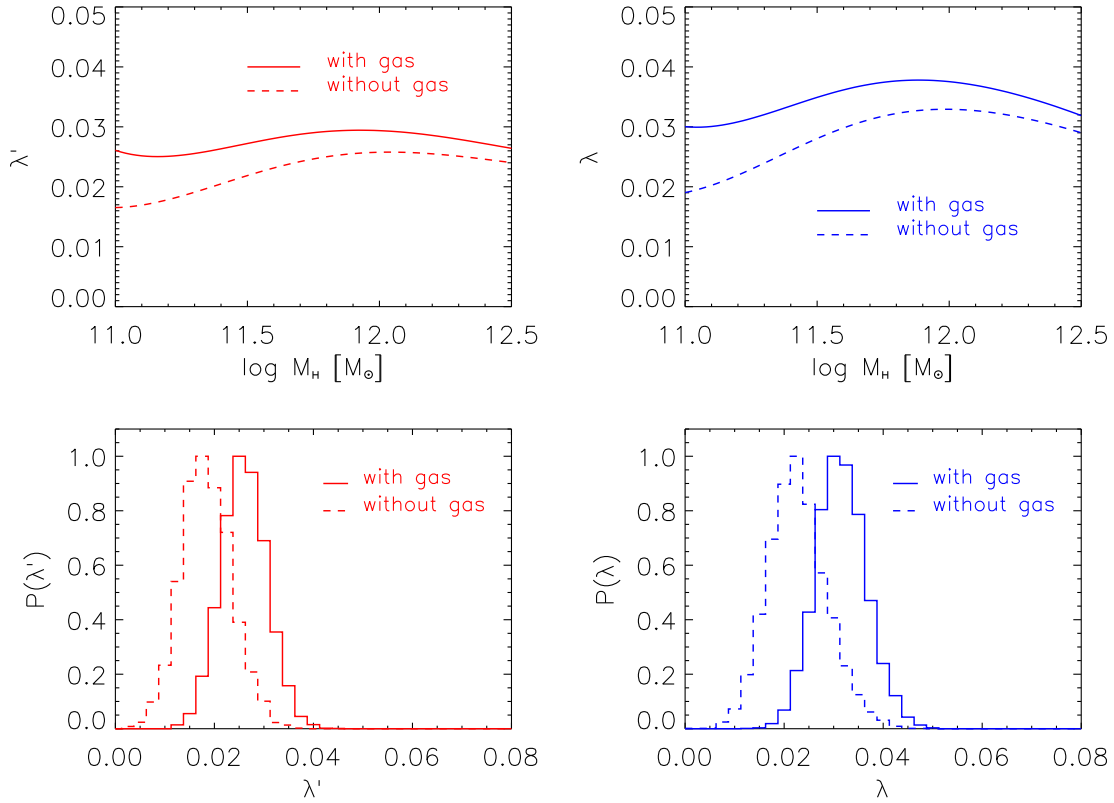


FIG. 2.— The spin parameter and its distribution function. *Top panels:* λ' (left) and λ (right) as a function of the halo mass, when the gas component is included in the system (solid line) and when it is not (dashed line). *Bottom panels:* the distribution function of λ' (left) and λ (right), again with gas and without gas.

& Burkert (2004) highlight that if one restricts one's attention to halos that hosts spirals and have not experienced major mergers during the late stages of their evolution ($z \lesssim 3$), the average spin parameter λ' turns out to be around 0.023, very close to our observational result. Moreover, Gardner (2001) and Peirani et al. (2004) showed that the spin parameter λ undergoes different evolutions in halos that have grown up mainly through major mergers or smooth accretion: in the former case λ takes on values around 0.044, while in the latter

case λ has lower values around 0.03.

Thus our findings point towards a scenario in which the late evolution of spiral galaxies may be characterized by a relatively poor history of major merging events.

We thank L. Danese, G. Gentile, and G.L. Granato for critical reading, and the referee for her/his helpful comments. This work is supported by ASI, INAF and MIUR.

REFERENCES

- Bullock, J.S., et al. 2001, *ApJ* 555, 240
 Burkert, A.M. & Salucci, P. 2000, *ApJ* 537, L9
 Burkert, A.M. & D'Onghia, E. 2005, in *Penetrating Bars Through Masks of Cosmic Dust*, ed. Block et al. (Dordrecht: Kluwer)
 Corbelli, E. & Salucci, P. 2000, *MNRAS* 311, 411
 Courteau, S., et al. 2003, *ApJ*, 594, 208
 Dale, D.A., et al. 1999, *AJ*, 118, 1489
 Dame, T.M. 1993, *AIPC* 278, 267
 D'Onghia, E. & Burkert, A.M. 2004, *ApJ* 612, L13
 Fall, S.M. & Efstathiou, G. 1980, *MNRAS* 193, 189
 Fukugita, M., Shimasaku, K., & Ichikawa, T. 1995, *PASP*, 107, 945
 Gardner, J.P. 2001, *ApJ*, 557, 616
 Giovanelli, R., et al. 1997, *ApJ*, 477, L1
 Mo, H.J., Mao, S. & White, S.D.M. 1998, *MNRAS* 295, 319
 Navarro, J.F., Frenk, C.S., & White, S.D.M. 1997, *ApJ*, 490, 493
 Navarro, J.F. & Steinmetz, M. 2000, *ApJ* 538, 477
 Peebles, P.J.E. 1969, *ApJ*, 155, 393
 Peirani, S., Mohayaee, R., & de Freitas Pacheco, J.A. 2004, *MNRAS*, 348, 921
 Persic, M., Salucci, P. & Stel, F. 1996, *MNRAS* 281, 27
 Persic, M. & Salucci, P. 1999, *MNRAS* 309, 923
 Persic, M. & Salucci, P., 1990, *MNRAS*, 245, 577
 Shankar, F., Lapi, A., Salucci, P., De Zotti, G., & Danese, L., 2005, *ApJ*, submitted
 Simard, L., et al. 1999, *ApJ*, 519, 563
 Tonini, C. & Salucci, P. 2004, in *Baryons in Dark Matter Halos*, ed. R. Dettmar et al. (<http://pos.sissa.it>), 89
 van den Bosch, F., Burkert, A., & Swaters, R.A. 2001, *MNRAS*, 326, 1205
 van den Bosch, F.C., et al. 2002, *ApJ*, 576, 21
 Vitvitska, M., et al. 2002, *ApJ*, 581, 799